

*Different ways of annotating and
proving a quicksort with Jack*

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Introduction

- Goal : verify a [quicksort](#) written in Java annotated with [JML](#) with Jack
- [Static verification](#) of this program :
 - done in an interactive way with Coq.
- Motivation : need for some simple examples in Jack
 - Already done in ESC/Java2
 - Already done in Krakatoa
 - Now doing it in [Jack](#)

Jack

The [Java Modelling Language](#) (JML) is used to annotate the Java programs we want to verify with Jack.

- Uses a [weakest precondition calculus](#) to generate proof obligations
- The proof obligations are decomposed w.r.t. the different [execution cases](#)
- Fully integrated in [Eclipse](#) as a plugin, and use a [plugin architecture](#) to include new provers
- To prove the quicksort we used the [Coq Output](#) from Jack
- Coq plugin in can be both used to prove [automatically and interactively](#) the proof obligations generated by Jack

Coq with Jack

The files generated by Jack in Coq can be separated into 3 categories :

- The **prelude** containing the logic used :
 - A static part (jack.arith.v jack.references.v jack.tactics.v)
 - A dynamic part generated for a specified class (myClass.classes.v myClass.subtypes.v myClass.v)
- The **proof obligation** (1 file)
- A file containing some **custom tactics** written by the user

To edit the files we use the **CoqEditor** plugin, an integrated editor for Coq files in Eclipse.

General Plan

1. Annotating in a normal way
2. Using purity
3. Using 'native' types

Annotating in a normal way

1. The QuickSort algorithm
2. Annotations
3. Kind of proof obligations

The QuickSort algorithm...

```
private void sort(int lo, int hi) {
    int left, right, pivot;
    if( !(lo < hi)) return;
    left = lo;
    right = hi;
    pivot = tab[hi];
    while(left < right) {
        while((left < right) && (tab[left] <= pivot)) left++;
        while((left < right) && (tab[right] >= pivot)) right--;
        if(left < right){
            swap(left, right);
        }
    }
}
```

...in Java

```
swap(left, hi);  
if (left > 0)  
    sort(lo, left - 1);  
if (left + 1 < tab.length)  
    sort(left + 1, hi);  
}
```

- Property wanted : it sorts the array
- The permutation property is left out, we use just an [injection](#) property

Annotations (1)

Specifications of the main method :

```
/*@ requires (tab != null) && (0 <= lo) && (lo < tab.length) &&  
  @      (0 <= hi) && (hi < tab.length);  
  @ modifies tab[lo .. hi];  
  @ ensures (\forall int i, j; (lo <= i) && (i <= hi) ==> (lo <= j) && (j <= hi)  
  @      ==> (i < j) ==> tab[i] <= tab[j]) &&  
  @      (\forall int i; lo <= i && i <= hi; (\exists int j; lo <= j && j <= hi &&  
  @          \old(tab[j]) == tab[i]));  
  @*/  
private void sort(int lo, int hi)
```

Annotations (2)

Annotations of the loop invariant :

```
/*@ loop_modifies left, right, tab[lo..(hi - 1)];  
  @ loop_invariant (lo <= left) && (left <= right) && (right <= hi) &&  
  @   (\forall int m; (lo <= m) && (m < left) ==> tab[m] <= pivot)  
  @   && (\forall int n; (right < n) && (n <= hi) ==> pivot <= tab[n])  
  @   && tab[right] >= pivot &&  
  @   (\forall int i; lo <= i && i <= hi - 1;  
  @       (\exists int j; lo <= j && j <= hi && \old(tab[j]) == tab[i]));  
  @ decreases (right - left);  
  @*/
```

Kind of proof obligations

- Around 230 proof obligations (POs)
- 100 are solved automatically
- Difficulties :
 - the **loop termination** wasn't so trivial
 - the use of **ghost variables** was necessary
 - the proof can easily get **messy**
 - use of assertions
 - the verbosity of the annotations

Using purity

1. Purity in JML
2. Purity in Jack
3. Simplifyng pure methods
4. Specification macros, predicates for JML
5. A quicksort annotated with predicates

Purity in JML

Pure methods are methods you can **use in your specification** in JML.

- No 'visible' side effect
- A constructor can be pure

→ We'd like to keep track of the pure method name in the proof obligations

Purity in Jack

- In Jack constructors are not pure
- At first pure methods were directly unfolded within the proof obligations
- Now there is a couple definition / hypothesis

Definition myfun_1 : A1Type → A2Type → ResType → Prop :=
fun (h arg1 arg2 Res) => Context1 → Res = true. (* ResType = bool *)

Definition myfun_2 : A1Type → A2Type → ResType → Prop :=
fun (h arg1 arg2 Res) => Context2 → Res = false.

...

(* Used within a hypothesis : *)

... → myfun_2 this l_var1 Res_23 → Res_23 = true.

Specification macros, predicates in JML

- Pure methods can be used to do some specification macros :

```
/*@ requires true;  
  @ ensures \result == ((tab != null) && (i >= 0) && (i < tab.length));  
  @*/  
public/*@ pure @*/ boolean withinBounds(Object[] tab, int i) {  
    return (tab != null) && (i >= 0) && (i < tab.length);  
}
```

We'd like to prove lemmas on this method to ease the proofs :

→ directly define pure methods in Coq

Coq predicates in JML

How the predicates are defined for Jack :

- Within JML :

```
//@ public native boolean withinBounds(Object[] tab, int i);
```

- In the file userTactics.v :

Definition withinBounds :

```
(Reference → t_int → Reference) → Reference → t_int → bool :=
```

```
fun intelements tab value =>
```

```
  andb (notb (REFeq tab null))
```

```
  (andb (Zle_bool 0 value) (Zlt_bool value (arraylength tab))).
```


A quicksort annotated with predicates

All the important annotations are replaced by these predicates :

```
/*@ public native boolean is_sorted(int[] tab, int beg, int end);  
  @ public native boolean is_inf(int [] tab, int value, int beg, int end);  
  @ public native boolean is_sup(int [] tab, int value, int beg, int end);  
  @ public native boolean withinBounds(int[] tab, int i);  
  @*/
```

These predicates are indeed **not so useful** by themselves :

- They permit to have **clearer** annotations `withinBounds(tab, i)` instead of `(i >= 0) && (i < tab.length)`
- We can prove **properties on them** at the Coq level

→ Some properties are still hard to express (permutation on arrays)

Using native types

1. Native types
2. Model variables
3. Quicksort with model

Native types

- To express the permutation property :
 - Have list in Coq, not in JML (since there is a list library in Coq...)
- A new construct to directly use a Coq defined datatype :
`//@ public native class IntList`
In the file userTactics.v :
 - Definition** IntList : list t_int.
- The types are not some standard Java/JML class types :
 - it does not inherit from the class object
 - it is not an instance, etc...
 - more of a functional type : modifiers create new objects and are 'static'

A list library

We can now define a list library binded on some Coq types, to use in annotations :

```
/*@ public native class IntList {  
  @ public static native IntList create();  
  @ public static native IntList cons(int i, IntList c);  
  @ public static native IntList app(IntList c1, IntList c2);  
  @ public native boolean is_in(int i);  
  @ public native boolean equals(IntList i);  
  @ public native int length();  
  @ }  
@*/
```

→ Link it with the program

Model variables

- Model variables are specification variables **associated** with a program variable.
- The association is done through a translation function.
- In JML it has the following syntax :
 - `//@ model MyType modVar ;`
 - `//@ represents modVar ← P(progVar)`
- You cannot do a 'set' with these variables, i.e. you cannot **assign** them.
→ The model variable is considered to be modified at the same time the program variable it represents

POs with model

For the post-condition of each method which is `using progVar` :

- Jack adds the condition :
 $\exists v, v = P(\text{progVar})$
- Jack replaces each occurrence of `modVar` in the post-condition of the method by `v`

We can link naturally a `IntList` model variable with a variable of the program

Quicksort with model (1)

```
//@ model IntList list ;
//@ represents list ← IntList.toList(tab) ;

/*@ public native class IntList {
    @ public static native IntList create() ;
    @ public static native IntList cons(int i, IntList c) ;
    @ public static native IntList app(IntList c1, IntList c2) ;
    @ public native boolean is_in(int i) ;
    @ public native boolean equals(IntList i) ;
    @ public native int length() ;
    @ public static native IntList toList(int []tab) ;
    @ }
    @*/
```

Quicksort with model (2)

```
/*@ requires (tab != null) && (0 <= i) && (i < tab.length)
   @      && (0 <= j) && (j < tab.length);
   @ modifies tab[i], tab[j], list;
   @ ensures tab[i] == \old(tab[j]) && (tab[j] == \old(tab[i])) &&
   @      list.permutation( \old(list));
   @*/
public void swap(int i, int j) {
    int tmp;
    tmp = tab[i];
    tab[i] = tab[j];
    tab[j] = tmp;
}
```


Conclusion

- The quicksort was annotated in different ways, and a proof has been made in its entirety for the version without the predicates
- We can now use some Coq predicates in Jack's JML annotations
- The use of Coq predicates for model variables is still in developpement
→ We have done some refinement on the datatype level
- The next step would be to do it more on the program level
→ Maybe use the JML's model program construct

Download links

Jack :

`http://www-sop.inria.fr/everest/soft/Jack/`

CoqEditor (only works with the CVS version of Coq!) :

`http://www-sop.inria.fr/everest/soft/Jack/UpdateSite/`

eassert, quicksort :

`http://www-sop.inria.fr/everest/personnel/Julien.Charles/`