Dèmonstration automatique d'obligations de preuve

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Outline

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The Satisfiability Modulo Theories (*SMT*) problem

- Automatically discharging proof obligations amounts to checking the satisfiability of first-order formulae w.r.t. a background theory
  - first-order formulae encoding program + property
  - background theory encoding relevant aspects of the program (e.g., user-defined data types, memory model, type system)
- Since usual deduction techniques cannot solve interesting SMT problems, many SMT solvers are developed:
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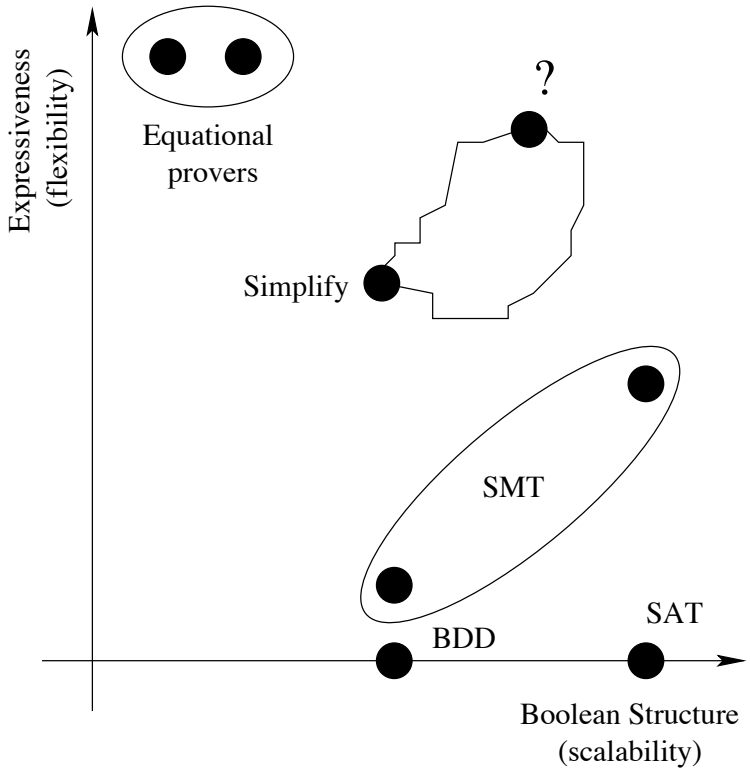
SMT solvers for Program Verification

Design SMT solvers with the following features:

- predictable: by reasoning modulo a decidable theory
- scalable: by carefully integrating efficient tools for handling various aspects of SMT pbs (e.g., Boolean structure, reasoning in the background theory, ...)
- expressive: by developing automated deduction techniques combining richer and richer theories (equality, fragments of Arithmetics, lists, arrays, pointers ...)

The situation (state-of-the-art) and beyond...



- sharp trade-off between expressiveness and scalability
- *Simplify* stands in-between but biased to expressiveness
- Our goal: design and implement new techniques for re-using available tools and ideas (SMT) to build satisfiability solvers ('?') offering a better trade-off between expressiveness and scalability, and providing us the certification of results for integration in larger (skeptical) verification tools

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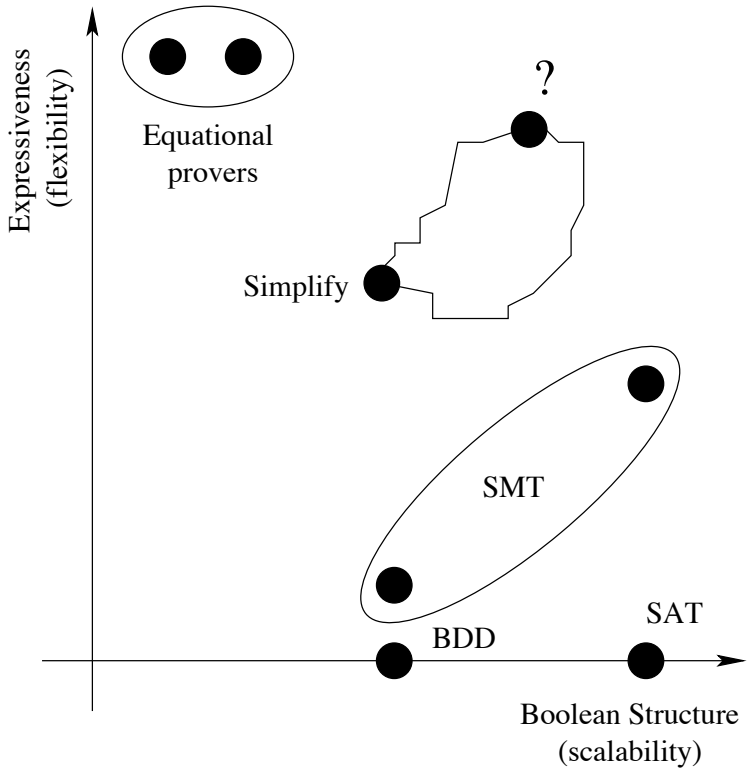

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haRVey architecture: basics (before GECCOO)

function Bool + *T* (ϕ: *ground formula*)
  $\phi^p \longleftarrow$ *nf*(*abs*(ϕ)) // *Abstract*
  while Bool-*satisfiable*(ϕ^p) **do begin**
    $\beta^p \longleftarrow$ *pick_prop_assignment*(ϕ^p)
    $(\rho, \pi) \longleftarrow$ *T-sat*(*prop2fol*(β^p)) // *Check*
    if $\rho \neq \bot$ **then return** *yes*
    $\phi^p \longleftarrow \phi^p \wedge \neg$*fol2prop*($\pi$) // *Refine*
  end
  return *no*
end

- π is the conflict set
- $\neg$*fol2prop*(π) is the conflict clause

A uniform methodology to design *T-sat*

- Superposition = refutation complete deduction rules for FOL with equality
- Terminates on interesting classes of formulae
- Provides rewrite-based decision procedures for free
  - works for data structures axiomatizations, e.g., uninterpreted function symbols, lists, arrays, and their combination *[ARR03]*
  - easy construction of conflict sets from proofs built by equational provers *[DR03]*
  - does not work for fragments of Arithmetics!

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Starting point: Nelson-Oppen combination schema

Nelson and Oppen's key idea: use satisfiability procedures as black boxes and synchronize them by exchanging a certain class of entailed formulae.

Strong Limitations:

- component theories must be signature-disjoint
- component theories must be stably infinite (i.e. satisfiable formulae have infinite models)

  ➽ not always the case, e.g. enumerated data types*!*
- Component decision procedures are considered as black boxes but we combine rewriting-based procedures with black box decision procedures...

  ➽ does this change the problem*?*

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Combination and Black Box Procedures

- Asymmetric extension of the N-O schema
  An extension of the Nelson-Oppen method for combining *polite* theories (subsuming stably infiniteness) and an arbitrary one (e.g., enumerated data types) ***[RRZ05]***

  ➽ beyond Stable Infinite
- Symmetric extension of the N-O schema
  An extension of the Nelson-Oppen method for combining theories in which satisfiability in arbitrary models and satisfiability in infinite models are decidable problems ***[BGN+06]***

  ➽ beyond Stable Infinite

Combination and Rewriting-based Procedures

- If Superposition is a decision procedures for T_1 and T_2, separately, it is so for the union of T_1 and T_2 under reasonable assumptions ***[ABRS05]***

  ➸ modularity of termination

- Rewriting-based methods for automatically checking whether a theory T is either stably-infinite ***[KRRT06]*** or admits only finite models ***[BGN$^+$06]***

  ➸ automatically checking Stable Infiniteness

- Rewriting-based methods for computing the entailed formulae to be exchanged by the Nelson-Oppen combination method ***[KRRT05]***

  ➸ deduction completeness

Decision Procedures for Data Structures

- Problem: data structures are often axiomatized with non-disjoint unions of arbitrary theories (i.e., possibly non-stably infinite)
- Our solution: new techniques (based on instantiation strategies) to avoid certain problems preventing the applicability of Nelson-Oppen method
  - decision procedures for various extensions of the theory of arrays (dimension, sortedness, ...) ***[GNRZ06]***

    ➸ beyond disjointness
  - decision procedure for a theory of pointers parametric in the theory of elements stored in memory cells ***[RZ06]***

    ➸ beyond disjointness and stably infiniteness

Integration of Boolean solvers and Decision Procedures

- Recall that a Boolean solver is available because of the SMT architecture...
- ***Key idea***: use the Boolean solver for the case-splitting on the derived disjunction of equalities between shared variables
- ***How***: every time a clause is derived by a procedure, pass it to the Boolean solver...
- For a similar approach (with less demanding requirements on the procedures being combined) see ***[BBC+05, BBC+06]*** on "*Delayed Theory Combination*"

What about certification ?

- New version of haRVey (based on the integration of a SAT solver and congruence closure) for proof reconstruction into ISABELLE ***[FMM+06]***
  - Combination of
    - ★ proof reconstruction from SAT solver
    - ★ proof reconstruction from congruence closure algorithm
- New prover: Ergo, based on a new combination method (congruence closure and Linear Arithmetic) for (partial) proof reconstruction in Coq
  - Neat and small architecture (3000 lines of Ocaml, close to inference rules)

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haRVey after GECCOO

- Two incarnations...
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  - haRVey-SAT (SAT, congruence closure, ...): in collaboration with P. Fontaine and S. Merz (Nancy, France)
  - Available at `http://harvey.loria.fr`
- Better trade-offs between expressiveness and scalability: haRVey-FOL
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Future work

- New combination methods: investigate relationships between politeness and rewriting-based procedures with the aim of finding automatic methods to characterize polite theories
- Heap allocated data structures:
  - generalize to arbitrary recursive data structures, such as trees, DAGs
  - incorporate resource reasoning (time and space consumption)
  - incorporate local reasoning *à la* Separation Logic
- Model checking infinite state systems: integration of standard algorithms for LTL model checking and (combination of) decision procedures for theories of data structures
- Standardization for interface and cooperation mechanisms of reasoning tools around the SMT-LIB initiative?

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