Using Automated Theorem Proving within the GECCOO Project

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Motivation

Theorem proving for real applications, e.g. JavaCard bytecode certification

Certicartes

Specification of Defensive VM (DVM) and Bytecode Verifier (BV)

- Specification of Offensive VM (OVM) and Abstract VM (AVM)
- Certification of i) OVM and ii) BV (AVM, monotonicity)

☞ by using the COQ Theorem Prover. Disadvantages:

- highly interactive
- script-based
- expensive error recovery

☞ by Automated Theorem Proving

Goal: Use this experience in the GECCOO project

Overview

I. The SPIKE Theorem Prover

- Specifications
- Inference System

II. Specification and Validation of JavaCard VMs

- Extensions of SPIKE
- Experimental Results

III. Working Plans

SPIKE: Settings

$$\left.\begin{array}{l}\text{axiomes } Ax\text{, conjecture } \phi \\ \text{relation de conséquence } \models\end{array}\right\} \; Ax \models^{?} \phi \quad \Leftrightarrow \quad Ax \models \phi\gamma$$

Problem: potential infinite number of closed instances $\phi\gamma$

☞ induction + reasoning techniques

Descente infinie" Principle

[Fermat (1659)]

$\mathcal{P}$ (infinite) set of formulas, $<$ Noetherian order over $\mathcal{P}$

1. for any counterexample from $\mathcal{P}$ there exists a smaller one

---

2. $\mathcal{P}$ has only true formulas

Applications :

- implicit induction provers
- saturation-based provers

Methodology for building provers

[Stratulat, 2000]

Sound and modular "descente infinie"-based theorem provers, providing:

✔ clear separation between logic and computation

✔ easy and sound extensions

✔ inference systems "à la carte"

SPIKE

✔ Specification

- first-order logic
- constructor-based many-sorted conditional specifications
- initial consequence relation
- syntactic order over the conditional equations

✔ Proofs

- implicit induction + deduction techniques (rewriting, subsumption, tautology elimination,...)

SPIKE: Specifications

```
specification : add
% problem description
sorts : nat ;
constructors :
0    :          → nat;
s_   : nat      → nat;
defined functions:
_+_ : nat nat → nat;
axioms:
0 + x = x;
s(x) + y = s(x+y);
% proof part
strategy: ...
conjectures: ...
```

SPIKE: The Inference System

- instance of an abstract inference system [Stratulat, 2000; Stratulat, 2001]
- deduction techniques

tautology elimination

elimination of subsumed clauses

C' subsumes C if $C'\sigma$ is a subclause of C

unconditional rewriting

$C[t]_p \rightarrow_{Ax\langle \mathsf{H} \rangle} C[t']_p$ iff $t \rightarrow_{Ax\langle \mathsf{H} \rangle} t'$ and $C[t']_p \prec C[t]_p$

case analysis

rules $\cup_{i=1}^{n} \{P_i \Rightarrow l_i \rightarrow r_i\}$ and $C[l_i\sigma_i]_{p_i}$

$\mathsf{CaseAnalysis}(C) \triangleq \cup_i \{P_i\sigma_i \Rightarrow C[r_i\sigma_i]_{p_i}\}$ if $Ax \models_{ini} \bigvee_i P_i\sigma_i$

SPIKE: Proofs

Ax axioms, E conjectures, H premises

inference rule $(E,\ H) \vdash (E',\ H')$

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inference rule $\quad (E,\ H) \vdash (E',\ H')$

derivation $\quad (E^0,\ H^0) \vdash \ldots \vdash (E^n,\ H^n) \ldots$

SPIKE: Proofs

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inference rule $(E,\ H) \vdash (E',\ H')$

derivation $(E^0,\ H^0) \vdash \ldots \vdash (E^n,\ H^n) \ldots \hookrightarrow^{E^n=\emptyset}$ success

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Properties

soundness

$(E,\ \emptyset) \vdash \ldots \vdash (\emptyset,\ H) \quad \Rightarrow \quad Ax \models E$

refutational soundness

$(E,\ \emptyset) \vdash \ldots \vdash (E',\ H) \qquad Ax \not\models E' \quad \Rightarrow \quad Ax \not\models E$

SPIKE: Proofs

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A derivation can

- terminate with success,
- terminate with failure (excerpt of a counterexample)
- diverge

SPIKE Inference System (official version)

PROTHEO group, LORIA [Bouhoula, 1997]

Generate $(E \cup \{C\},\ H) \vdash (E \cup (\cup_\sigma E_\sigma),\ H \cup \{C\})$
if for any test substitution σ of C

- either $C\sigma$ is a tautology and $E_\sigma = \emptyset$,
- or $C\sigma \rightarrow_{Ax\langle H \cup E \cup \{C\}\rangle} C'$ and $E_\sigma = \{C'\}$
- otherwise, $E_\sigma =$CaseAnalysis$(C\sigma)$

Case Simplify $(E \cup \{C\},\ H) \vdash (E \cup E',\ H)$
if $E' =$CaseAnalysis(C)

Simplify $(E \cup \{C\},\ H) \vdash (E \cup \{C'\},\ H)$
if $C \rightarrow_{Ax\langle H \cup E\rangle} C'$

Subsume $(E \cup \{C\},\ H) \vdash (E,\ H)$
if C is subsumed by another clause from $Ax \cup H \cup E$

JavaCard Specifications

Getting SPIKE specifications from COQ

The converter is written in OCAML using the $camlp4$ preprocessor.

COQ Specification

```
Definition CONV := [t,t':type_prim][state:jcvm_state]
Cases (stack_f state) of
(cons h lf) ⇒
  Cases (extr_from_opstack t (opstack h)) of
  (Some (k, lv)) ⇒ (update_frame (push_opstack (
           vPrim (tpz2vp t' (t_convert t t' k))) lv h) state) |
  None ⇒ (AbortCode opstack_error state)
  end |
_ ⇒ (AbortCode state_error state)
end.
```

OCAML Specification

☞ COQ extraction commands

```
let cONV t t' state =
  match stack_f state with
    Nil → abortCode State_error state
  | Cons (h, lf) →
      (match extr_from_opstack t (opstack h) with
         Some p →
           (let Pair (k, lv) = p in
           update_frame
             (push_opstack (VPrim (tpz2vp t'
                (t_convert t t' k))) lv h) state)
       | None → abortCode Opstack_error state)
```

SPIKE Specification

☞ the converter OCAML $->$ SPIKE

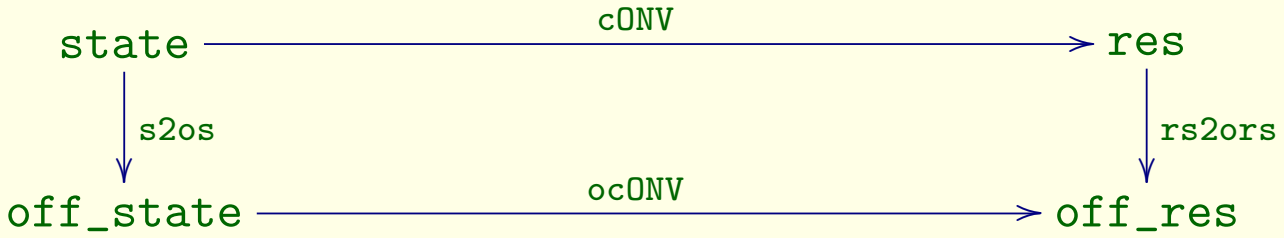
```
axioms : stack_f( state) = Nil
         ⇒ cONV( t, t', state) = abortCode( State_error, state);

         stack_f( state) = Cons( h, lf),
         extr_from_opstack( t, opstack( h)) = Some( Pair( k, lv))
         ⇒ cONV( t, t', state) = update_frame( push_opstack(
           VPrim( tpz2vp( t', t_convert( t, t', k))), lv, h), state);

         stack_f( state) = Cons( h, lf),
         extr_from_opstack( t, opstack( h)) = None
         ⇒ cONV( t, t', state) = abortCode( Opstack_error, state);
```

DVM - OVM Commutation Proofs

Input: offensive and defensive machines

Output: each instruction commutes (45 instructions)

```
conjectures :
        state = Build_jcvm_state( sh, hp, Cons (h, lf)),
        res = cONV(n, z0, state) ⇒
        res = abortCode( Type_error, state),
        res = abortCode( Signature_error, state),
        rs2ors( res) = ocONV (n, z0, s2os(state));
```



Proofs by:
- case analysis
- induction

Features

- sufficient complete and ground convergent
- parameterized specification (lists, pairs)
- existential variables (OCAML local variables)
- "big" specification

  61 sorts, 164 constructors, 446 defined functions, $\sim$ 2200 rules (February 2002)

SPIKE Improvements: Existential Variables

```
stack_f( state) = Cons( h, lf),
extr_from_opstack( t, opstack( h)) = Some( Pair( k, lv))
 ⇒ cONV( t, t', state) = update_frame( push_opstack(
    VPrim( tpz2vp( t', t_convert( t, t', k))), lv, h), state);
```

1. conditional equations cannot be oriented into rewrite rules

   a) use premises only in "safe" proof fragments

2. Generate failure

$$eq(O,O) = True \qquad eq(O,S(x)) = False$$
$$eq(S(x),O) = False \quad eq(S(x),S(y)) = eq(x,y)$$

   ☞ $eq(a,O)$ cannot be instanciated

   a) generalization of existential variables in universal variables

   b) definition changement, then case analysis:

$$x = O \wedge y = O \Rightarrow eq(x,y) = True \qquad x = O \wedge y = S(z) \Rightarrow eq(x,y) = False$$
$$x = S(z) \wedge y = O \Rightarrow eq(x,y) = False \quad x = S(z) \wedge y = S(t) \Rightarrow eq(x,y) = eq(z,t)$$

SPIKE Improvements: Dealing with "Big" Specifications

Need of automatic and efficient operations:

- extraction of subspecifications for each instruction
- recording of previous failed operations
- "fast" versions of Generate and Case Simplify
- specific simplification techniques

New Generate Rule

☞ test substitutions by unification

Generate $(E \cup \{C\},\ H) \vdash (E \cup (\cup_\sigma E_\sigma),\ H \cup \{C\})$

if $\exists t \in C$

- having induction variables and
- $\exists a \in Ax$ defining $t\#head$ and $(\sigma, \sigma_a) = mgu(t, a)$ and
- $C\sigma[t] \rightarrow_{\{a\sigma_a\}} C'$ and $E_\sigma = \{C'\}$

```
%  **** l_update_nth ****
l_update_nth( Nil, n, a) = Nil;
l_update_nth( Cons( x0, xs), 0, a) = Cons( a, xs);
l_update_nth( Cons( x0, xs), S( n'), a) = Cons( x0, l_update_nth( xs, n', a))
```

$l\_update\_nth(u_1, u_2, u_3)$ has the test substitutions

$$\{\langle u_1 \leftarrow Nil\rangle, \langle u_1 \leftarrow Cons(u_4, u_5), u_2 \leftarrow O\rangle, \langle u_1 \leftarrow Cons(u_4, u_5), u_2 \leftarrow S(u_6)\rangle\}$$

☞ faster (σ and $a\sigma_a$ is directly computed from mgu)

☞ less new conjectures ($\sim$ 10 w.r.t. $\sim$ 1000 for the old version)

New Case Simplify Rule

☞ no $\bigvee_i P_i\sigma_i$ test for terms with no induction variables.

Example:

```
P(0) = True     ⇒ f(0) = True;
P(S(x)) = True  ⇒ f(S(x)) = True;
P(x) = False    ⇒ f(x) = False;
```

☞ no application on $f(x)$

Specific Simplification Techniques

- congruence closure on conditions

  $(E \cup \{a = b \wedge \ldots \wedge b = c \wedge \ldots \Rightarrow \ldots\}, H) \quad \vdash$

  $(E \cup \{a = b \wedge \ldots \wedge b = c \wedge \ldots \wedge a = c \Rightarrow \ldots\}, H)$

- auto simplification with rules $e \to t$

  $(E \cup \{e = t \wedge \ldots \wedge s[e] \wedge \ldots \Rightarrow \ldots\}, H) \quad \vdash$

  $(E \cup \{e = t \wedge \ldots \wedge s[t] \wedge \ldots \Rightarrow \ldots\}, H)$

- augmentation with lemmas $C_1 \Rightarrow C_2$

  $(E \cup \{C_1' \Rightarrow C_2'\}, H) \quad \vdash (E \cup \{C_1' \wedge C_2\sigma \Rightarrow C_2'\}, H)$

  if C_1 is subsumed by $C_1' \Rightarrow C_2'$ with substitution σ

Experiments

Proofs [Barthe, Stratulat 2003]

- DVM - OVM: 41 (21 zero-knowledge proofs)
- DVM - AVM: 17
- Monotonicity: 21

Interaction of the user with SPIKE by

- fixing a precedence over the function symbols (for Generate and rewriting)
- providing intermediate lemmas (needs deep knowledge of the problem)
- changing the proof strategy (very few cases)
- changing function definitions (very few cases)

Conclusions wrt Certicartes project

- industrial applications with "home-made" automated theorem provers
  - access to internal structure
  - shaping according to needs (by using the building methodology)
- successful use of SPIKE
  - first full first-order specification for JavaCard platform
  - VMs certification
  - detection of errors and excerpt of counterexamples
  - high degree of automatization
  - efficiency ($<$ 4 hours for the DVM-OVM proofss on a machine with two 1GHz Intel Pentium III processors $+$ 896 Mbytes RAM)
- dramatic time reduction in the design of certified specifications

Working Plans

☞ following a similar working methodology for the GECCOO case studies.

SPIKE improvements

- more user interaction (treatment of difficult cases)
- more convivial user interface (graphical interface, shorter proof traces)

Integration of SPIKE with other GECCOO tools

- haRVey ?
- SPIKE plugin for ECLIPSE

Automated Theorem Proving for JavaCard Bytecode Certification

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References

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Content of the talk